

Math 241

Winter 2024

Lecture 18

Unit Circle
Sin, Cos, Tan

Feb 19-8:47 AM

Rectangular Coordinate System vs Polar Coordinate System

(x, y)

(r, θ)

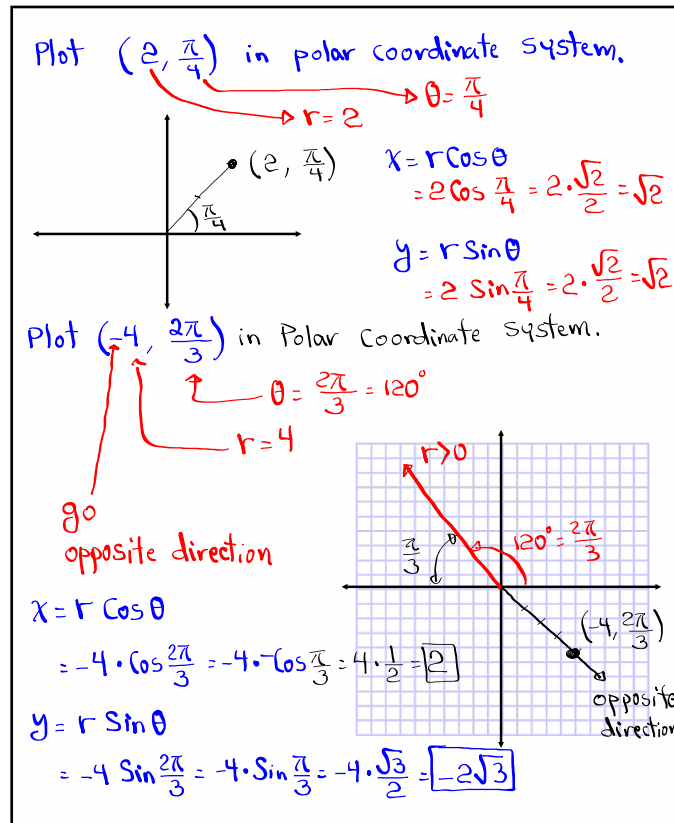
$$r = \sqrt{x^2 + y^2}$$

$$x = r \cos \theta$$

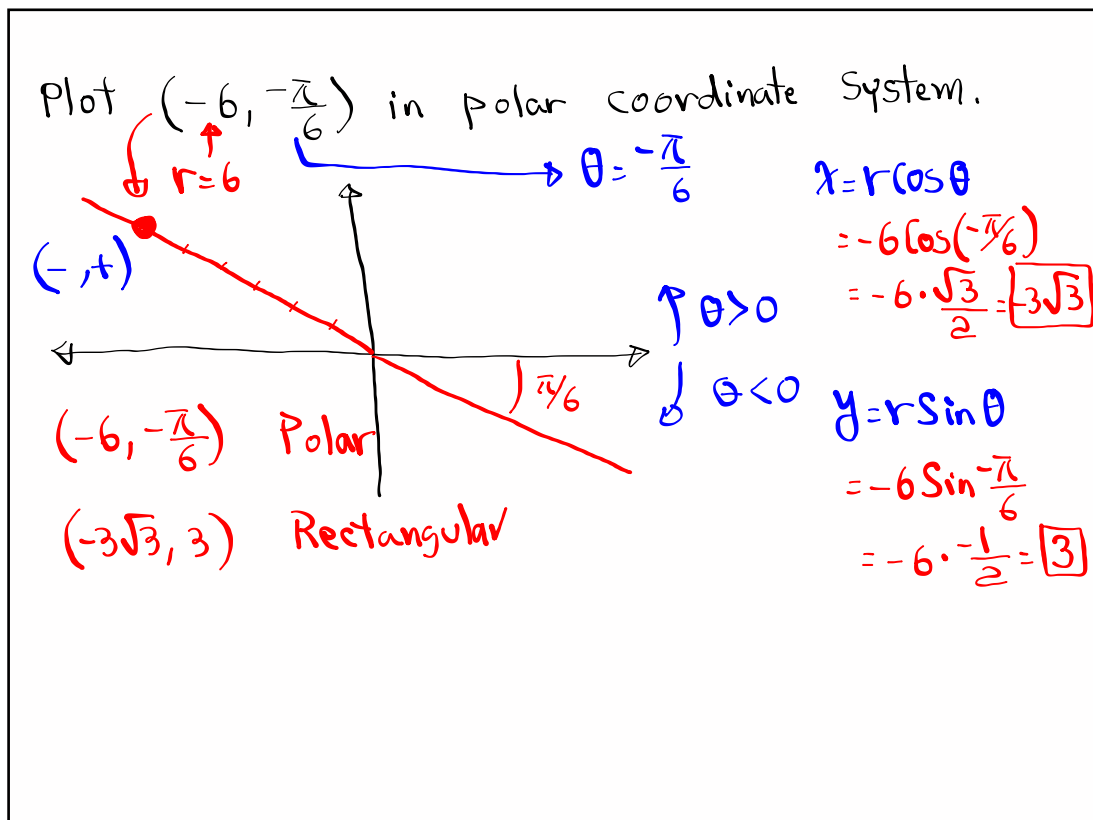
$$y = r \sin \theta$$

$$\tan \theta = \frac{y}{x}$$

Feb 1-8:09 AM



Feb 1-8:14 AM

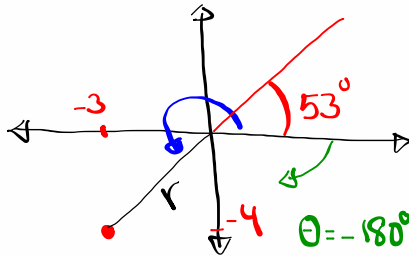


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Find Polar Coordinates for the point $(-3, -4)$
in rectangular.

$$r = \sqrt{x^2 + y^2} = \sqrt{(-3)^2 + (-4)^2} = \sqrt{25} = 5$$

$$\boxed{r=5}$$



$$\tan \theta = \frac{y}{x} \quad \tan \theta = \frac{-4}{-3}$$

$$\theta = -180^\circ + \tan^{-1} \frac{4}{3} = -180^\circ + 53^\circ = -127^\circ$$

$$RA = \tan^{-1} \frac{4}{3} \approx 53^\circ$$

$$\theta = 180^\circ + RA$$

$$\theta = 180^\circ + 53^\circ \approx 233^\circ$$

$(-3, -4)$

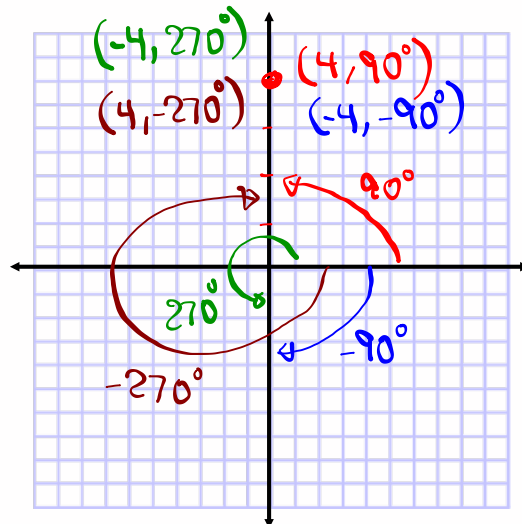
$(5, 233^\circ)$

$(-5, 53^\circ)$

$(5, -127^\circ)$

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Plot $(4, 90^\circ)$, $(-4, -90^\circ)$, $(-4, 270^\circ)$, $(4, -270^\circ)$



Feb 1-8:35 AM

Consider $(-2, 2)$ in rectangular

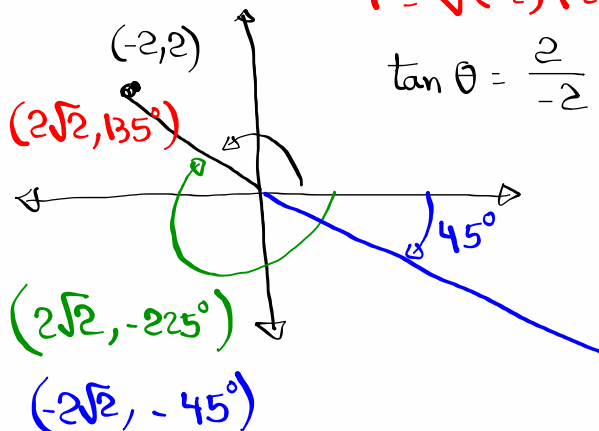
$$r = \sqrt{(-2)^2 + 2^2} = \sqrt{8} = 2\sqrt{2}$$

$$\tan \theta = \frac{2}{-2}$$

$$\tan \theta = -1$$

$$\text{RA } 45^\circ$$

$$\theta = 180^\circ - 45^\circ = 135^\circ$$



Feb 1-8:39 AM

Graphing Simple Polar equations:

Graph $r = 4$

$$\sqrt{x^2 + y^2} = 4$$

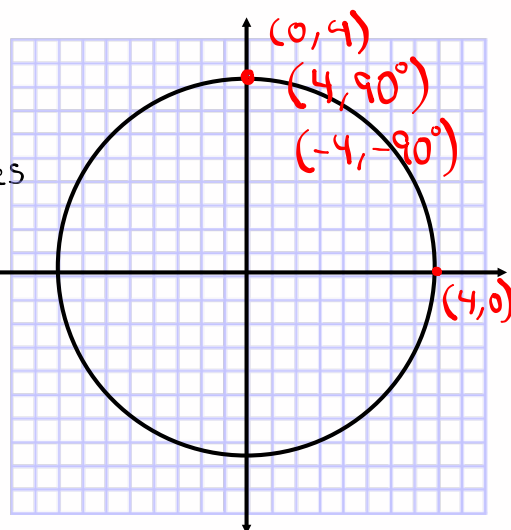
Square both sides

$$x^2 + y^2 = 16$$

Circle

Center $(0, 0)$

Radius 4



Feb 1-8:44 AM

Graph $r = \frac{6}{2 \cos \theta + 3 \sin \theta}$

Cross-Multiply $2r \cos \theta + 3r \sin \theta = 6$

Convert to rectangular

$2x + 3y = 6$

x	y
0	2
3	0

Feb 1-8:48 AM

Graph $r = 2 \sin \theta$

Multiply both sides by r

$r^2 = 2r \sin \theta$

Convert to rectangular

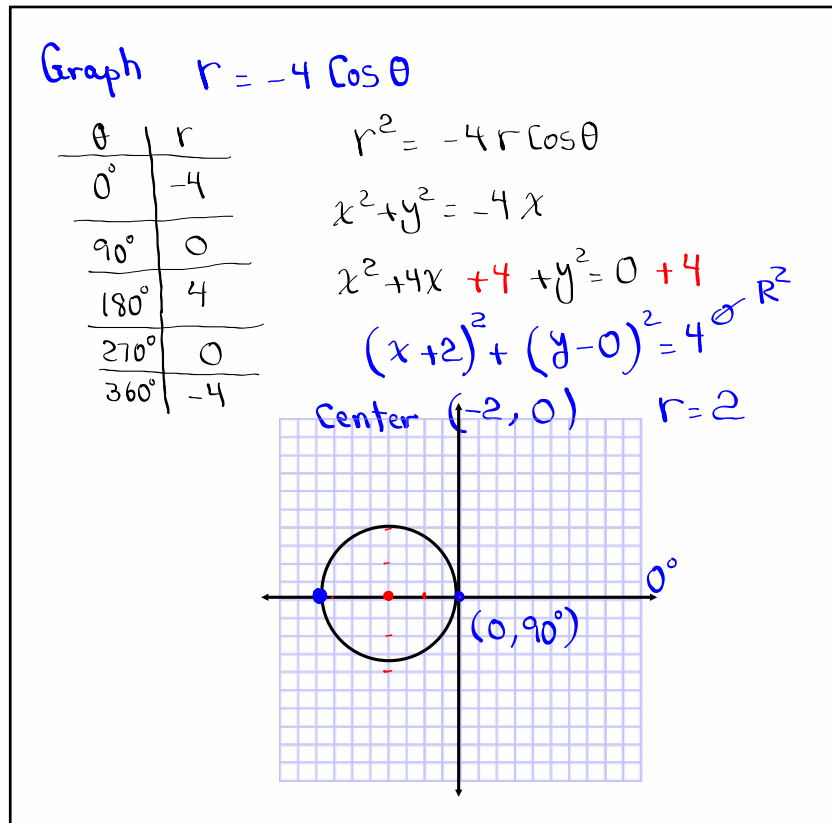
$x^2 + y^2 = 2y$

$(x-h)^2 + (y-k)^2 = R^2$ $x^2 + y^2 - 2y + 1 = 0 + 1$

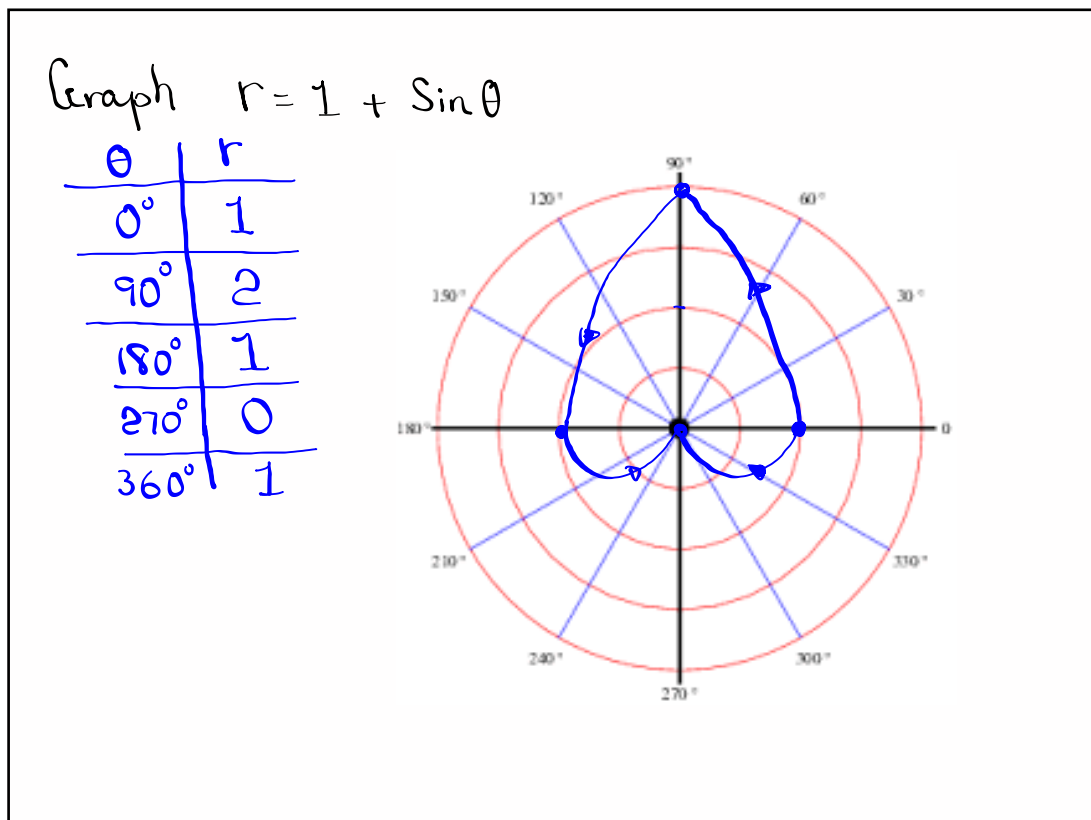
$(x-0)^2 + (y-1)^2 = 1$

Circle
Center $(0, 1)$ Radius 1

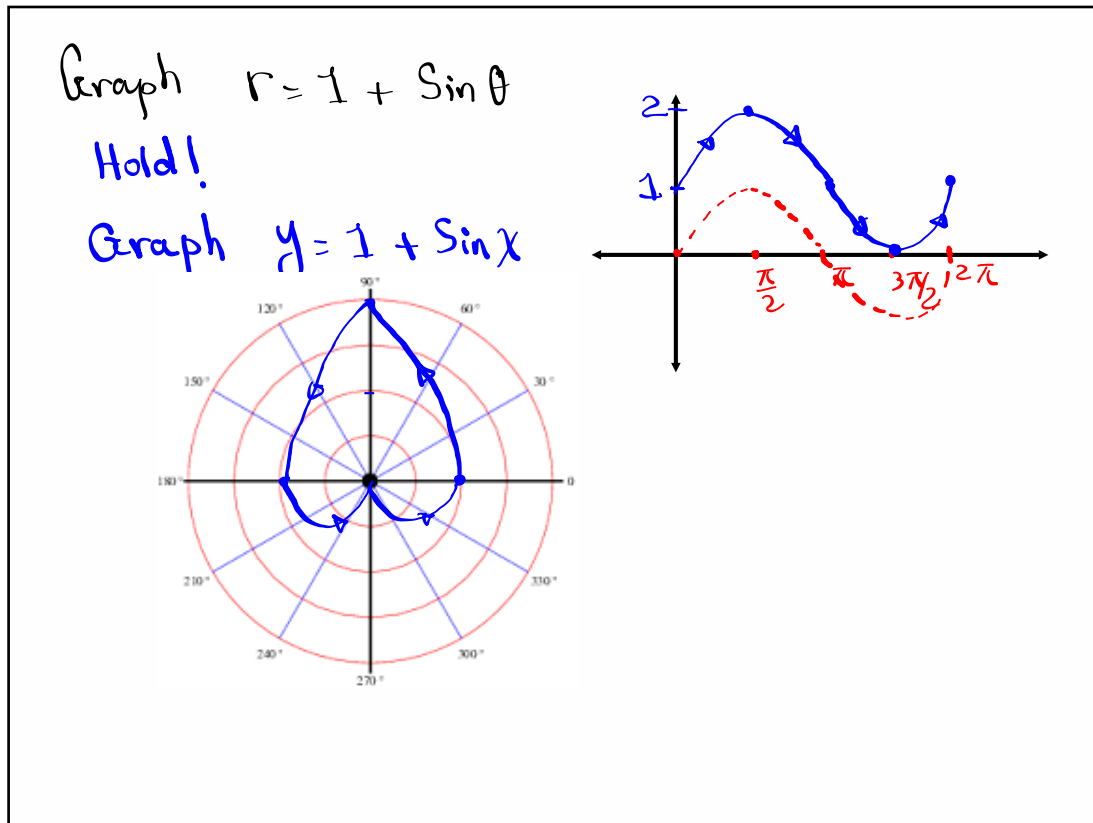
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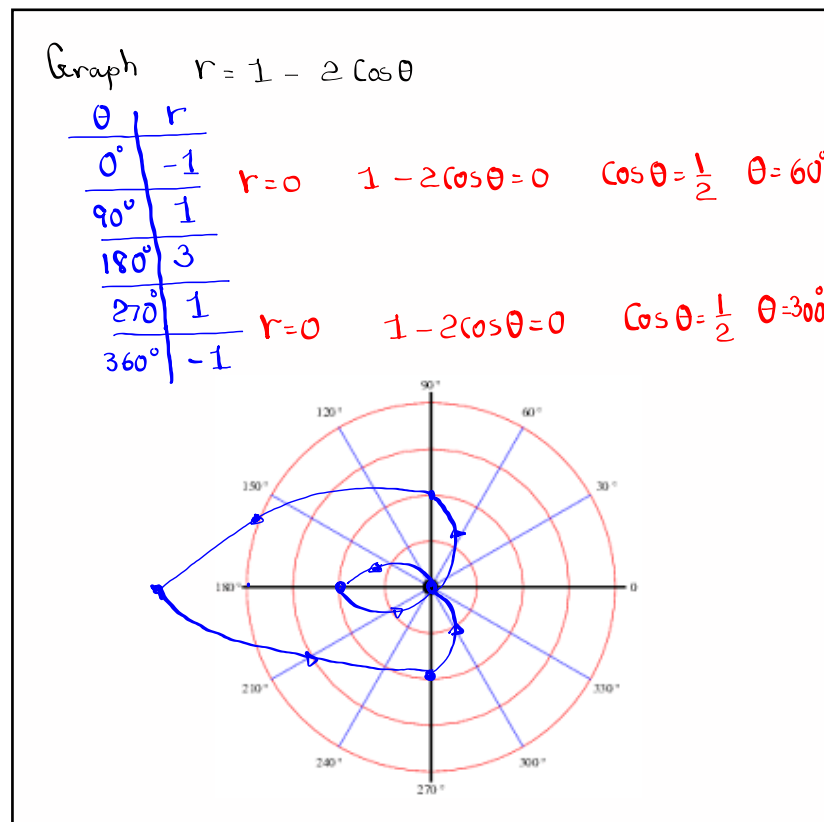
Feb 1-9:00 AM



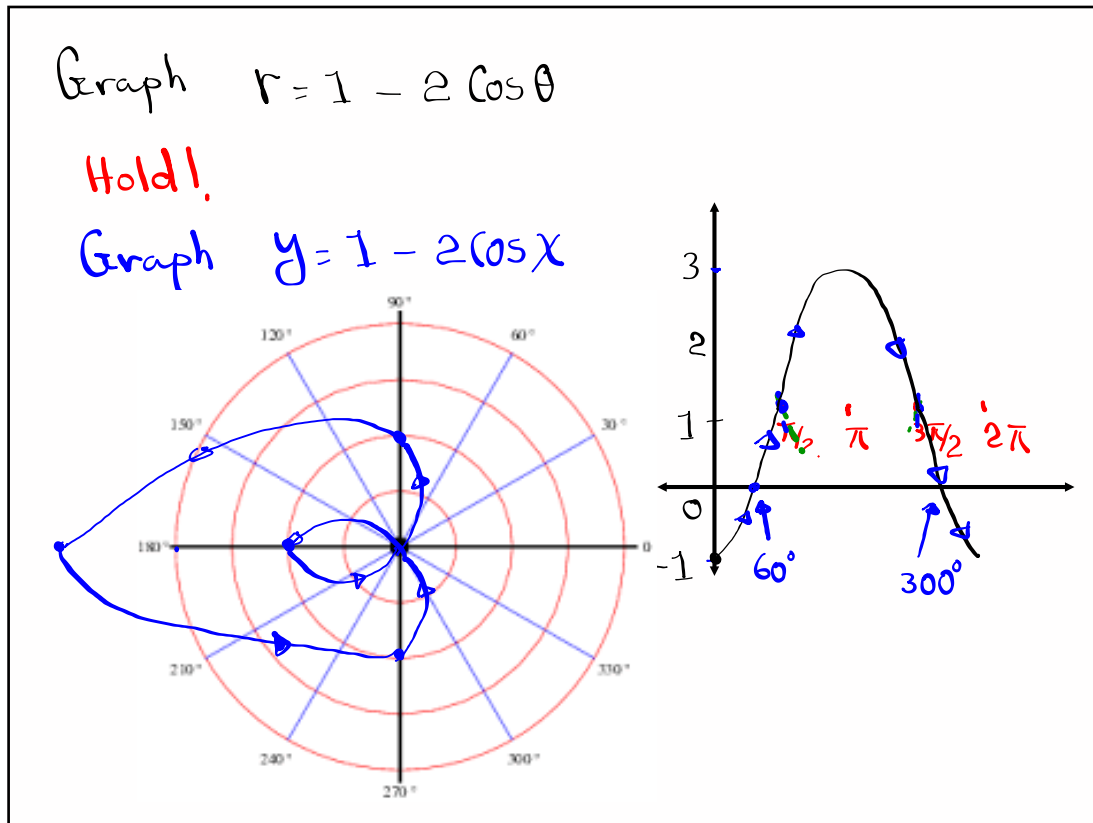
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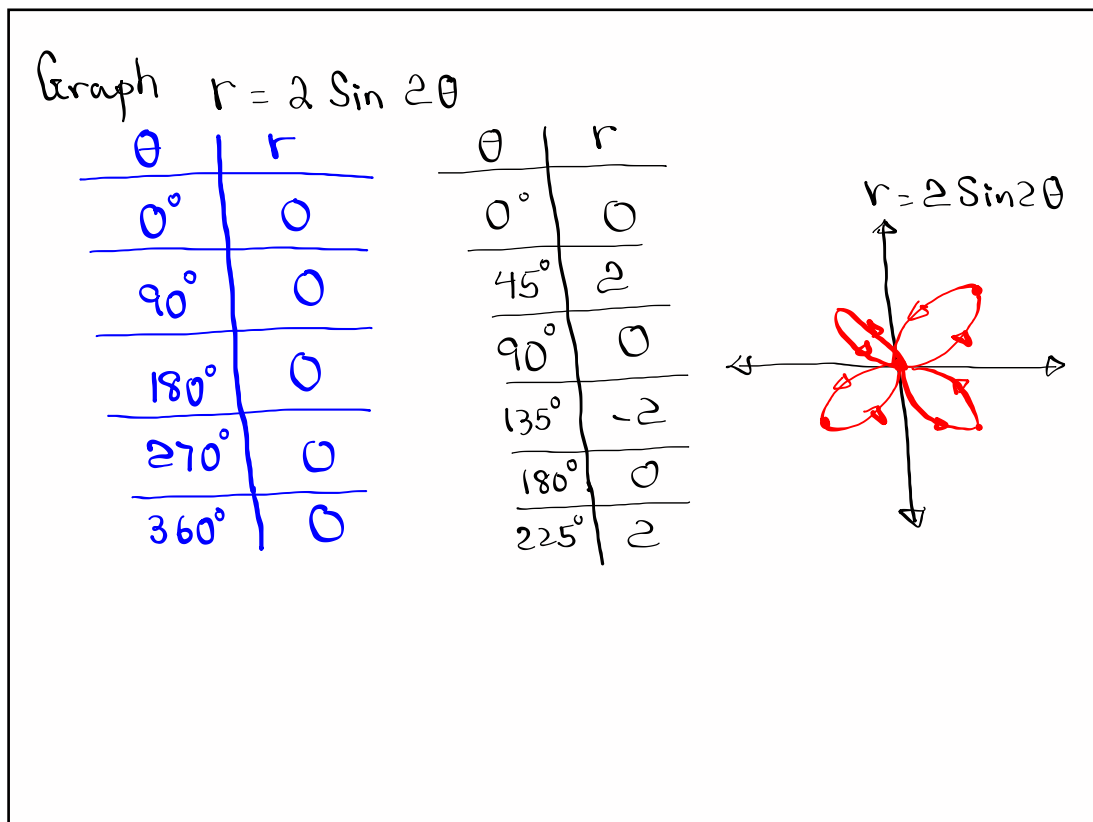
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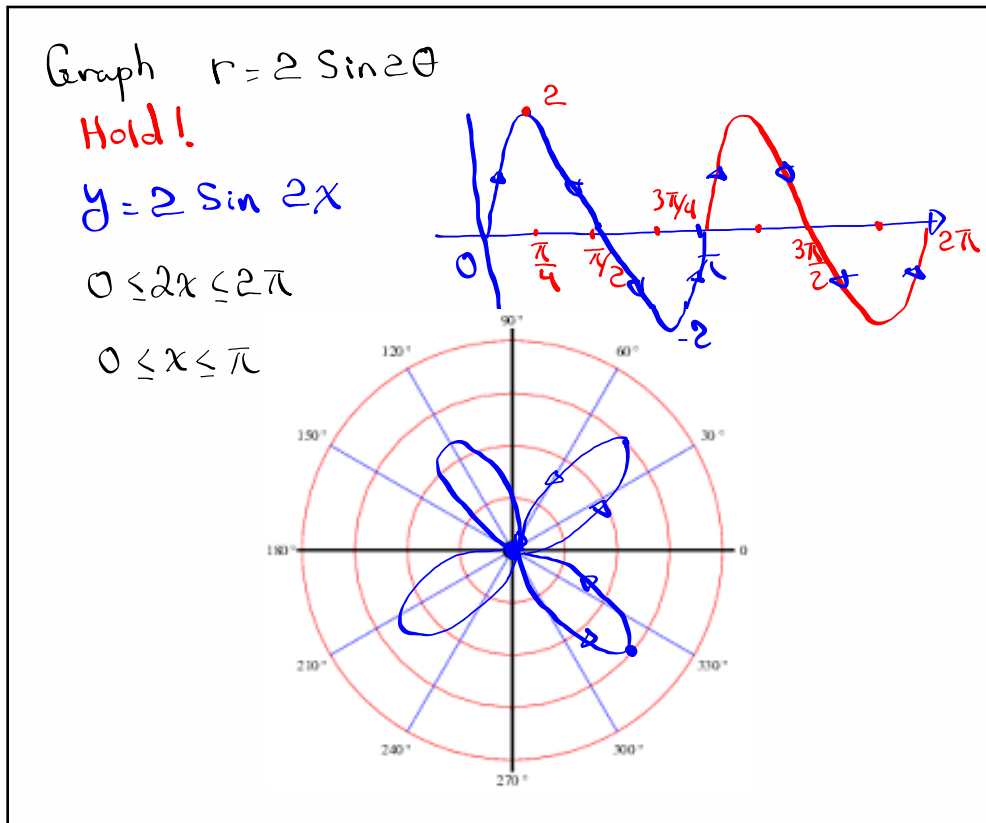
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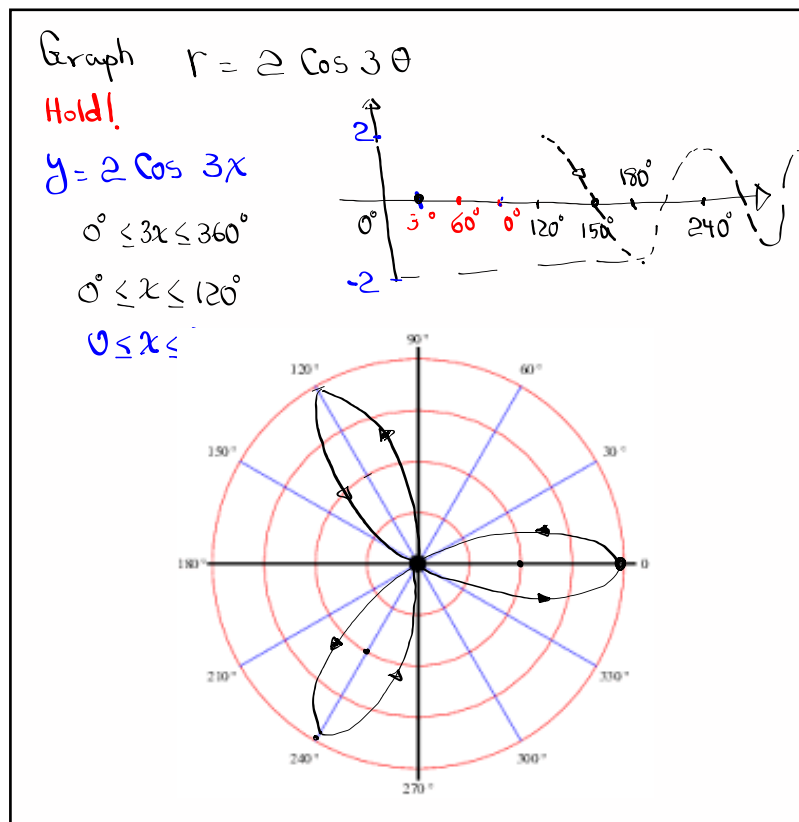
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Feb 1-9:56 AM



Feb 1-10:04 AM



Feb 1-10:09 AM

$$r = a \sin n\theta$$

$$r = a \cos n\theta$$

Roses

Loops \rightarrow Petals

If n is even $\rightarrow 2n$ petals

If n is odd $\rightarrow n$ petals

$r = 3 \cos 5\theta$ $\rightarrow 5$ is odd $\rightarrow 5$ petals

$r = 2 \sin 4\theta$ $\rightarrow 4$ is even $\rightarrow 8$ petals

Feb 1-10:19 AM

Graph $r^2 = \sin 2\theta$

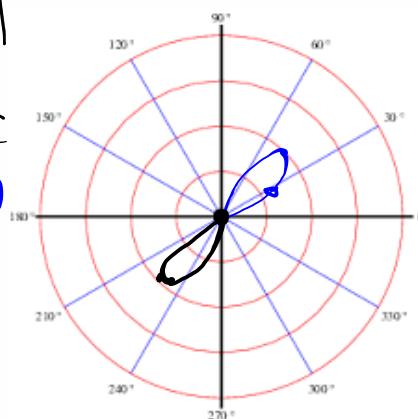
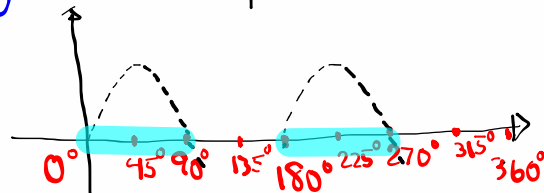
Hold

$$y^2 = \sin 2x$$

$$y = \pm \sqrt{\sin 2x}$$

Can we take square-root of negative number? **NO**

Graph $\sin 2x$



Feb 1-10:23 AM

